Further Mathematics HL

Overall grade boundaries

Higher level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 23	24 - 33	34 - 46	47 - 58	59 - 71	72 - 100

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 53	54 - 70	71 - 87	88 - 104	105 - 150

The areas of the programme and examination which appeared difficult for the candidates

On this paper candidates found difficulty with Apollonius' circle theorem, Ceva's theorem, vector spaces and probability generating functions.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on mean and variance of linear combinations of n random variables, Diophantine equations, recurrence relations and the Euclidean algorithm.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

The question caused a number of problems for candidates. In part (a) a number of candidates thought part (i) was correct as they did not realise it was an element and a number thought part (ii) was correct as they did not recognise 57 as a prime number. In both of these two cases, candidates then suggested part (iii) was false giving a variety of incorrect justifications. Part (b) was more successful for most candidates with many wholly correct answers seen. Part (c) again saw many correct answers, but some candidates tried to argue the opposite, incorrect viewpoint or in other cases gave no reason for their decisions, showing a complete misunderstanding of the command term “determine”.

Question 2

This was one of the more successful questions on the paper with many wholly correct answers seen. Only a very small number failed to complete part (a) successfully. There were also many fully correct answers to part (b). Part (c) caused a problem for some candidates where in most of those cases they failed to calculate the variance correctly.

Question 3

This was also a very successful question with many wholly correct answers seen. A small number of candidates made arithmetic errors in the calculations. Some candidates used unnecessarily long and complex methods for parts (b) and (c) which would have potentially left them short of time elsewhere.

Question 4

Most candidates had an understanding of how to start the question, but only a small number were able to gain full marks. It appeared that many candidates were used to finding p-values, but showed a lack of understanding when asked to find the critical regions and test a t-value. The conclusions required in part (d) were often too brief and/or poorly expressed.

Question 5

This question caused a problem for many candidates and only a small number of fully correct answers were seen. Most candidates were able to find a generalised equation of a tangent, but were then unable to see what could be replaced in order to find a quadratic equation that could be solved.

Question 6

This was a successful question for many candidates with many wholly correct answers seen. The vast majority of candidates were able to answer parts (a) and (b) and most knew how to start part (c). A few candidates were let down by not realising the need for a degree of formality in the presentation of the inductive proof.

Question 7

Again this was a reasonably successful question for many candidates with full marks often being awarded. However a significant minority were let down by giving very informal and descriptive answers which only gained partial marks. As the command term in the question is “prove” there is a need for a degree of formality and an explicit use of limits was expected.

Question 8

This question was the first one on the paper to cause a significant problem for the majority of candidates. Many were unable to start and a small number were unable to successfully deal with the algebraic manipulation required from the method they had embarked upon. For those who were successful at part (a), part (b) was often not fully correct, again due to the degree of formality required from the command term “prove”.

Question 9

This was a successful question for many students with many wholly correct answers seen. Part (a) was successfully answered by most candidates and those candidates usually had a reasonable understanding of how to complete part (b). A number were not fully successful in knowing how to explain their results.

Question 10

Many candidates were able to make a beginning to this question and attempted a solution to part (a). Some were let down by being unable to fully explain their reasoning. In part (b) a number of fully correct answers were seen but some candidates appeared to be completely unaware of what constituted a sensible approach. Again in part (c) a number of correct answers were seen but a significant number also appeared to have little idea on how to start.

Question 11

This was again a question which a significant number of students were unable to start. For those who did start only a small number understood the significance of “if and only if” meaning that wholly correct answers were not often seen.

Question 12

Again this was found difficult by many candidates and resulted in no attempt being made. For those who were able to start, parts (a) and (b)(i) showed a reasonable degree of understanding. After that it was only a significant minority of candidates who were able to proceed successfully with many ignoring or not realising the significance of the word “orthogonal”.

Question 13

Again this was a question that tested candidates and although many started only a very limited number were able to make significant progress. Part (a) was rarely done well with most candidates not understanding what was required. There was a little more success with part (b) but a number of candidates attempted methods that were not going to lead to anything meaningful. Most candidates did not understand what was required from part (c) and few correct answers were seen, even taking into account the fact that follow through marks could be awarded from (b).

Question 14

This was a more successful question for many candidates with a number of fully correct solutions being seen and a significant number of partially correct answers. Most candidates understood what was required from part (a)(i), but part (ii) often resulted in unnecessarily complex algebra which they were unable to manipulate. Part (b) resulted in many wholly successful answers, provided candidates realised the need for care in terms of the manipulation.

Recommendations and guidance for the teaching of future candidates

- It was pleasing to see a number of candidates make good and meaningful attempts at all questions, showing a good overall knowledge of the whole syllabus. However, many scripts were seen where candidates would answer questions on specific topics almost wholly correctly and then other questions would not be attempted at all, suggesting they were relying on the option they have covered for Mathematics HL with a small amount of information on the other topics. Unless all six topics are covered fully it is unlikely that candidates will be successful.
- A number of students were let down by not appreciating the level of formality and precision needed in terms of what they write. Within a Further Mathematics HL course this is a requirement.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 47	48 - 67	68 - 88	89 - 108	109 - 150

General comments

Overall, the paper appeared to be well-received by candidates, with many candidates displaying good knowledge across all areas of the syllabus. Indeed, the majority of candidates attempted most questions. However, a small number of candidates, had clearly not prepared much beyond one or two Mathematics HL options.

The areas of the programme and examination which appeared difficult for the candidates

- The Central Limit Theorem.
- Linear recurrence relations.
- Most candidates were unable to use a Maclaurin series for a numerical calculation to a required degree of accuracy.
- The cycle notation for permutations.

The areas of the programme and examination in which candidates appeared well prepared

- Properties of graphs, and the use of algorithms.
- Repeated use of l'Hôpital's rule.
- Simple coordinate geometry of a circle and a line.
- The solution of differential equations.
- The axioms of group theory.
- The construction of a Maclaurin series.
- Matrix multiplication.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

(a) and (b) were generally well done. A few candidates said that the graph was not Eulerian because it contained more than two odd vertices. A few candidates failed to back up their assertion that the graph was Hamiltonian by stating an example of a relevant cycle. In part (c) some candidates did not clearly indicate that they had used Kruskal's algorithm, but just drew a minimum spanning tree.

Question 2

In part (b) the infinite upper limit was rarely treated rigorously. In answering part (c) many failed to say that the Central Limit Theorem is valid for large samples and for any initial distribution. The parameters of the distribution were often not stated.

Question 3

This question was usually well done, using a variety of valid approaches.

Question 4

Although (a), (b) and (c) were generally well done, it was rare to see a completely satisfactory geometrical answer to part (c)(ii). A few candidates solved the differential equation as a homogeneous equation.

Question 4

For part (d) most candidates used the correct solution method for a homogeneous differential equation. A few found the algebra hard going in finding the particular solution. Most approaches to the final part were unsatisfactory, with a lack of proper consideration of the inequalities in the question.

Question 5

A significant number of candidates had clearly not learned the mechanical procedure for solving linear three-term recurrences. Those who were well prepared, coped well with parts (a) and (c). Part (b) was very rarely successfully answered. Some candidates proved that $v_{n+1} > v_n$ but erroneously concluded that the sequence diverged.

Question 6

Parts (a), (b) and (c) were generally well done. In a few cases, squaring a general element was thought, erroneously, to be sufficient to prove closure in part (a). In part (d) closure was rarely established satisfactorily. Part (e) was often tackled well.

Question 7

Part (a) was generally answered, albeit either with an excess of algebraic manipulation or with too little – candidates need to realise that when an answer is given in the question, they need to convincingly reach that answer. In part (b)(i), the results of part (a) were well used for up to the quadratic term. The obtaining of the cubic term, and more so the quartic term, was often not convincing. In part (ii), poor communication let down many candidates. In answering part (iii), many candidates failed to realise that in order to prove the stated inequality, they needed to actually write down the number 1.025354…, which is clearly greater than 1.02535.

Question 8

In part (a)(i), many just wrote down $n!$ without showing how this arises by a sequential choice process. Part (ii) was usually correctly answered, although some gave their answers in the unwanted 2-dimensional form. Part (iii) was often well answered, though some candidates failed to realise that they need to explicitly evaluate the product of two elements in both orders. Part (b) was often well answered. A number of candidates found 2x2 matrices – this gained no marks. Nearly all candidates knew how to approach part (c)(i), but failed to be completely convincing. Few candidates seemed to know that every permutation can be written as a product of non-overlapping cycles, as the first step in part (ii).

Recommendations and guidance for the teaching of future candidates

The Further Mathematics syllabus covers very many aspects of mathematics from the mechanical use of standard methods to very abstract and formal logical arguments. Teachers should emphasise the importance of candidates setting out their work in a logical fashion and that all relevant working needs to be shown clearly. Students need to be aware that questions may test different aspects of several options within a single framework.